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EXERCISE ON PLURALS

In order to deal with plurals, philosophers and linguists have proposed to introduce a new kind of object in our ontology: besides (singular) individuals, truth values and sets/characteristic functions of them, we have plural individuals or individual sums.[1] We can form plural individuals by conjoining existing individuals with the "individual sum" operation $\cup_i$. The set D_e now contains singular/atomic individuals and plural/non-atomic individuals, ordered by the reflexive, anti-symmetric and transitive "part-of" relation $\leq_i$.[2]

(1) $\{a,b\} \neq a\cup_i b$

(2) $a \leq_i a\cup_i b\cup_i c$

(3) ⟦ [$_{DP}$**Pat, Mary and Susan**] ⟧ $= p\cup_i m\cup_i s$

This exercise is concerned with plural definite descriptions, e.g. **the children**. The final aim of this exercise is two-fold: (i) to decide on a denotation for plural Nouns (e.g., **children**), derivable from the standard denotation of the corresponding singular Noun; and (ii) to define a denotation for the definite article **the** that derives the right presuppositions and truth conditions for both singular and plural definite NPs. To accomplish these goals, you will be guided part of the way.

(i) *Plural Nouns*. In mereological PrL, the operators "*" and "$^{\otimes}$" apply to 1-place predicates to yield the kinds of denotations exemplified in the left column in (4). For the time being, consider the two possible denotations of the NatLg **children** given in the right column. Also, keep in mind the equivalences under (5).

(4)

Mereological PrL	NatLg
a. ⟦CHILD⟧^{s12} = $\{a, b, c\}$	a'. ⟦**child**⟧^{s12} = char$\{a, b, c\}$
b. ⟦*CHILD⟧^{s12} = $\{a, b, c, a\cup_i b, a\cup_i c, b\cup_i c, a\cup_i b\cup_i c\}$	b'. ⟦**children**⟧^{s12} =? char $\{a, b, c, a\cup_i b, a\cup_i c, b\cup_i c, a\cup_i b\cup_i c\}$
c. ⟦$^{\otimes}$CHILD⟧^{s12}= $\{a\cup_i b, a\cup_i c, b\cup_i c, a\cup_i b\cup_i c\}$	c'. ⟦**children**⟧^{s12} =? char $\{a\cup_i b, a\cup_i c, b\cup_i c, a\cup_i b\cup_i c\}$

(5) a. If P denotes a singleton, ⟦*P⟧ = ⟦P⟧.
b. If P denotes a singleton, ⟦$^{\otimes}$P⟧ = $\varnothing$.

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[1] See the mereological version of Predicate Logic in Leonard-Goodman 1949 and Goodman-Quine 1947.

[2] The join operation $\cup_i$ has the properties of idempotency, commutativity, associativity and absorption (see Partee-ter Meulen-Wall p. 279-80). That is, the so-enlarged D_e is a complete join semilattice.

(ii) ***Plural the***. The intuitions that we have to capture are illustrated under (6): plural definite descriptions carry a non-uniqueness presupposition, and they denote the maximal individual of which the plural predicate holds. Two informal trials, corresponding to the possibilities (4b'-c'), are sketched under (7)-(8).

(6) **The children are tired.**
a. Presuposition: There is more than one child (in situation s).
b. Denotation of the DP: the maximal plural individual in $[\![\textbf{children}]\!]^s$;
e.g. $[\![\textbf{the children}]\!]^{s12} = a \cup_i b \cup_i c$.

(7) Possibility 1: (4b'). [Informally]
$[\![\textbf{The}_{+PL}]\!]^s$ = $\lambda X_{<et>}$: there is a non-atomic y_e such that X(y)=1 .
the maximal individual z_e for such that X(z)=1

(8) Possibility 2: (4c'). [Informally]
$[\![\textbf{The}_{+PL}]\!]^s$ = $\lambda X_{<et>}$: $X \neq char_{\varnothing}$.
the maximal individual z_e for such that X(z)=1

(iii) Here comes your task. Contrary to **the**$_{+PL}$, we saw in class that **the**$_{+SING}$ has a uniqueness presupposition and it denotes that unique individual. The informal version of **the**$_{+SING}$ is given under (9). Your task is to define one single denotation for **the** that will derive the right presuppositions and denotations no matter whether its Noun is plural or singular. In doing so, you'll have to decide between * and $^{\otimes}$ as the denotation of NatLg plural marking. (You can do (10) informally first, and then formalize it by using mereological PrL+λ+ι.)

(9) $[\![\textbf{The}_{+SING}]\!]^s$ = $\lambda X_{<et>}$: there is exactly one x for which X(x)=1 .
the unique y such that X(y)=1.

(10) $[\![\textbf{The}]\!]^s$ =